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Estimation of Fuzzy Sets for Computational Colour Categorization

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Abstract: Colour is an important visual cue for computer vision applications. However, up to the present, the automatic assignment of names to image regions has not been widely used due to the non-existence of a general computational model for colour categorization. In this paper we present a model for colour naming based on fuzzy set theory, in which each of the 11 basic colour terms defined by Berlin and Kay[1] is modelled as a fuzzy set with a characteristic function which assigns a membership value to the category to any colour sample. The model is based on combining two well known functions, a sigmoid and a gaussian, to define a membership function for colour categories. It is denoted here as the sigmoid-gaussian function and it fulfils a set of properties which makes it adequate to this purpose. The characteristic functions for each colour category have been fitted to data obtained from a psycho-physical experiment and the model has been tested on the Munsell colour array to show its validity. The results obtained indicate that our approach can be very useful as a first step to expand the use of colour naming information in computer vision applications.

Key words: Colour categorization; computational models; computer vision; fuzzy sets

INTRODUCTION

Colour naming, or colour categorization, has been studied from different disciplines. For many years, biologists, philosophers, linguistics and anthropologists have worked about this topic providing very different points of view about the problem. Recently, Schirillo[2] has presented an extensive and complete review of this topic. To understand the motivation of our work we will summarize those works which are closer to our goal, that is, the computational automation of the colour naming task in the frame of computer vision applications.

The basis of most of the works on colour naming has been the study of Berlin and Kay[1] in which they stated the existence of a unique and common set of eleven basic colour terms in different languages. Several later works have confirmed this result from experiments on the OSA space[3,4], the Munsell space[5,6] and also when comparing the colour naming in Japanese and American observers.[7]

In the way of explaining the colour naming process, Kay and McDaniel[8] proposed a general model of colour naming which attempted to find the relationship between the neuro-physiological mechanisms involved in colour naming and the semantic categories of basic colour terms. The model is based on fuzzy set theory where each colour category has a characteristic function which defines a membership degree to the category. The most interesting of the model is that it considers the colour naming problem as something more than the assignment of a colour term to a stimulus, since the fuzzy approach takes into account the non-discrete nature of the problem.

Afterwards, some works have studied the structure of the colour naming space[9,10,11] and some models[12,13,14] have been proposed. In a different point of view, some studies about the dependence of the colour naming task to the illumination changes have also been presented.[15,16]

In computer vision, colour is a very important visual cue for image understanding and it has been used to perform very different tasks such as object recognition,[17] image segmentation,[18] image indexing[19] or tracking.[20] However, the automatic assignment of names to image regions has not been widely dealt up to now, although it could be very useful for some automatic visual tasks such as image annotation, image indexing, object recognition, or robotics. Up to the present, Lammens[21] has been the only one in proposing a parametrical model for colour naming in different colour spaces. In this model, each colour category is modelled by a variant of the gaussian function which is fitted on the Munsell colour space considering the boundaries and focuses for each category defined by Berlin and Kay for American English. The model obtains interesting results, but it has not been extensively tested.

Hence, we can state that although colour naming has been widely studied from different disciplines, the computational automation of this task is still in its first stages. It seems evident that a computational model allowing several degrees of membership to the colour categories is the most adequate for this goal. Moreover, if we want to automate colour naming with the same behaviour as a human being, we have to use a learning set of data obtained from human observations as the basis of our model. To this end, in this paper, we present a general computational colour naming model in which each one of the eleven basic colour categories is modelled by a fuzzy set with a characteristic function. The goal of the model is, for a given stimulus, to assign the same colour name that a human observer would do. In the next sections, we firstly present a fuzzy set framework for colour naming. Secondly, we build a computational learning set from a psycho-physical colour naming experiment, and we use this set to fit the membership functions of each fuzzy set corresponding to the colour categories. The model is tested on the Munsell colour array used by Berlin and Kay and our automatic categorization of this space is compared to previous results. Finally, we present the conclusions of this work and future research lines.

FUZZY SETS FOR COLOUR NAMING

The final goal of this work is to build a computational model which allows defining a decision function that automatically performs the colour naming visual task. Our model is based on the idea proposed by Kay and McDaniel[8] which considers the colour naming task as a fuzzy decision. The best way to mathematically model this decision functions is by considering the basis of the fuzzy set theory.[22]

A fuzzy set is described by its membership function. In colour naming, we can consider that any colour category, C_k, is a fuzzy set with a membership function, f_{C_k}, which assigns to each colour sample $\underline{x}$ a membership value $f_{C_k}(\underline{x})$ within the [0,1] interval. This value represents the certainty we have about $\underline{x}$ has to be named with the linguistic term, t_k, corresponding to category C_k. From this point of view, the first step of any colour naming modelling process will be the definition of the membership functions for each colour category. Once these functions are defined, it will be possible to compute a colour descriptor, $CD(\underline{x})$, such as:

$$CD(\underline{x}) = (f_{C_1}(\underline{x}),...,f_{C_n}(\underline{x})) = (m_1,...,m_n) \quad \text{where} \quad m_k \in [0,1] \;\; \forall k = 1,...,n \quad \text{and} \quad \sum_k m_k = 1 \qquad (1)$$

$CD(\underline{x})$ describes the membership relation of $\underline{x}$ to each colour category, m_k is the certainty value associated to $\underline{x}$ by f_{C_k} and n is the number of categories considered. In our case n=11 and the categories considered in the model are the corresponding to the 11 basic colour terms proposed by Berlin and Kay, that is $t_k \in$ {white, black, red, green, yellow, blue, brown, purple, pink, orange, and grey} $\forall k = 1,...,n$. Therefore the colour descriptor $CD(\underline{x})$ defined above is a vector of 11 components and the information contained in such descriptor can be used by a decision function to decide the colour name of a given stimulus $\underline{x}$.

The goal of the next sections will be to estimate the memberships functions of the colour categories, f_{C_k}. To this end, we will firstly build a learning set and, secondly, we will define the shape of these functions. Afterwards, we will estimate their parameters according to the psycho-physical data.

BUILDING THE LEARNING SET

The fuzzy approach to the problem presented in the previous section requires a proper learning set as the basis to develop the colour naming model. To this end, we need to perform two steps. Firstly, we have to select a set of surfaces with their computational representations and, secondly, we have to collect the name assignments performed by human observers to these samples. To achieve these goals, 422 different colour samples were selected and their reflectances were measured and tabulated using a PhotoResearch PR-650 spectro-radiometer. These reflectances were selected trying to cover a region of the colour space as wide as possible and have been the basis to build two sets of data. The first set is a computational representation of these reflectances in a 3D colour space, and the second is a set of colour naming assignments to the samples obtained from a psychophysical experiment. In the next paragraphs we go deeply in these two points.

A computational colour space

In computer vision the starting point is normally a digital image acquired with a camera. Cameras are usually based on the use of prisms to decompose the input signal into a set of signals only containing wavelengths that correspond to each one of the camera sensors. Normally, three sensor devices are used. The three sensors provide a decomposition of the input signal into three channels: R (red), G (green), and B (blue). Therefore, the colour space generated by the camera is a device-dependent RGB space. Computational colour is normally based on the dichromatic model of Shafer[23] which allows to consider the tri-stimulus integration as:

$$x_i = \int E(\lambda) S(\lambda) R_i(\lambda) d\lambda \qquad (2)$$

where x_i represents the *i*-th component of vector $\underline{x}$=(R,G,B), that is, the RGB response corresponding to a point in the scene with $S(\lambda)$ as the surface reflectance function, $E(\lambda)$ as the spectral distribution of the illuminant of the scene and $R_i(\lambda)$ as the spectral sensitivity of the *i*-th sensor of the device. In our case, we have used the sensitivities of a common camera (SONY DXC-930) obtained from the Computational Colour Vision Laboratory of the Simon Fraser University.[24] The camera response was white balanced according to the selected illuminant conditions (D65 daylight) in order to match the white reflectance to the maximum value represented in each colour channel of the image. The original and the balanced sensitivities are shown in figure 1. Thus, the RGB values of the selected reflectances were computed according to equation 2.

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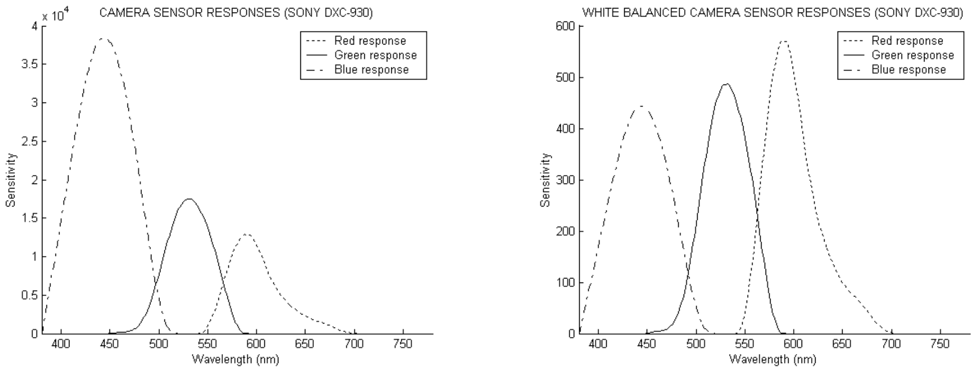

FIG. 1. Sensitivities of the SONY DXC-930 camera used to generate the RGB values of the colour samples selected to build the learning set. Left: Original sensitivities of the camera. Right: White balanced camera responses.

At this point, one possibility would be to transform the RGB values to a standard colour space, such as CIE XYZ, CIELuv, CIELab, etc. However, in computer vision, we will normally work under uncontrolled conditions and the acquisition parameters will be unknown. To obtain the CIE XYZ values from the RGB of a camera we should know the spectral response of the camera which is not normally provided by the manufacturer. Another possibility would be to obtain the camera response by using a monochromator, but it should be done on strictly controlled lab conditions which is not practical for most industrial applications of computer vision. Hence, our computational model must be able to work on a device-dependent colour space. However, in this work, data analysis will be also performed on the CIELab space for comparative purposes. Experimentation on how our model behaves when the acquisition device is changed is left for further works.

To establish our colour naming model based on a specific parametric function that we will introduce afterwards, we need to separate chromaticity information from intensity. The initial RGB space is transformed to a new 3D-space, denoted here as *uvI*, where the first two axes represent chromaticity and the third one corresponds to the intensity. To obtain the chromaticity information of axes *u* and *v*, the colour vectors are projected on the unit plane and referred to the *uv* coordinate system. The intensity axis, *I*, corresponds to the module of the original RGB vectors. This space is defined by equations 3 and 4. In figure 2, an scheme of the transformation from RGB to *uvI* space is shown.

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 2^{-1/2} & 0 & -2^{-1/2} \\ 0 & \frac{6^{1/2}}{2} & 0 \end{pmatrix} \begin{pmatrix} r \\ g \\ b \end{pmatrix} + \begin{pmatrix} 2^{-1/2} \\ 0 \end{pmatrix} \tag{3}$$

$$I = \left(R^2 + G^2 + B^2\right)^{1/2} \tag{4}$$

where

$$r = \frac{R}{R+G+B}, \qquad g = \frac{G}{R+G+B}, \qquad b = \frac{B}{R+G+B} \tag{5}$$

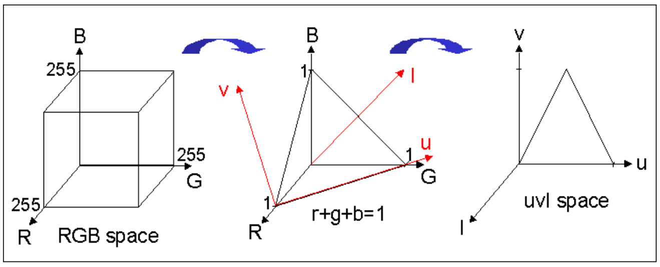

FIG. 2. Scheme of the transformation from the RGB colour space to the *uvI* space.

A colour naming experiment

To collect the naming assignments, we made a psychophysical experiment.[25] Since we have considered colour naming as a fuzzy problem, a simple colour naming experiment in which a colour term is assigned to each sample is not enough to build our set of naming assignments. Hence, in our experiment subjects were asked to distribute 10 points between the eleven basic colour terms according to the certainty they had about the sample belonging to different categories. Thus, if the subject was absolutely sure about the colour name of a sample, then the 10 points had to be assigned to the category corresponding to that name. Otherwise, if there was a doubt between two or more names, the 10 points had to be distributed between the categories corresponding to those names. The constraint of only using the eleven basic colour terms was set to reach a high degree of consistency and consensus, which is highly desirable for our purpose. Previous experiments[4,6,9] have shown that basic colour terms are used more consistently and with greater consensus than non-basic names.

The experiment was developed under a D65 illuminant and using the set of 422 colour samples mentioned above. The samples were presented in random order, one at a time, to 10 subjects with normal colour vision. All the subjects were screened for colour vision deficiencies using the Ishihara test. The experiment was done twice by all 10 subjects and in the second trial the samples were presented in reverse order. This implied a total number of 8440 observations. For each sample, the scores were averaged and normalized to the [0,1] interval. Further information on the procedures and details about the experiment have been explained in a previous work[25].

After this experiment, we have all the necessary data to build a proper learning set. On one hand, we have a three-dimensional representation of a set of 422 different colours, and on the other hand, we have their corresponding fuzzy assignment performed by human observers corresponding to the set of eleven colour categories. In the next section we present the process followed to estimate the underlying statistical model of these data.

DEFINING THE MEMBERSHIP FUNCTIONS

Once the learning set has been built, we are ready to propose a general colour naming model. The first step is to define the membership functions of the colour fuzzy-sets. The next step is to estimate the parameters of these functions for each colour category by using the learning set built in the previous section. This implies to solve a non-linear data-fitting problem in the least squares sense, that is, minimizing the mean squared error (MSE) between the membership values provided by the model and the values obtained in the experiment. For each of the 11 colour categories, the fitting was done using a Large-Scale optimization algorithm which minimizes the following expression:

$$\min_{\theta_k} \frac{1}{2} \sum_{j=1}^{s} \left(f_{C_k}\left(\underline{x}^j, \theta_k\right) - CD_k(\underline{x}^j) \right)^2 \tag{6}$$

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where s is the number of samples in the learning set, C_k is the colour category being modelled with $k=1,...,n$, $\underline{x}^j$ is the j-th sample of the learning set, $CD_k(\underline{x}^j)$ is the k-th component of the colour descriptor obtained from the experiment for the sample $\underline{x}^j$, and θ_k is the set of parameters of the membership function f_{C_k}. Now, the problem is to decide which is the best function to represent the colour name membership. We will examine two different functions: a simple gaussian function and a sigmoid function modulated by a one-dimensional gaussian.

Gaussian model

This first approach is based on the work of Lammens[21] that assumes gaussian membership functions given by:

$$f_{c_k}(x,\theta_k) = G(\underline{x},(\underline{\mu},\Sigma)) = e^{\frac{-1}{2}(\underline{x}-\underline{\mu})^T \Sigma^{-1}(\underline{x}-\underline{\mu})} \quad (7)$$

where $\theta_k=(\underline{\mu},\Sigma)$. The fitting of this functions to the learning set was done both in our *uvI* space and in the CIELab space. The results of these fittings in terms of the MSE obtained for the 11 colour categories are presented in table I. The sum of the individual MSE's obtained for this fitting on the *uvI* space was 14.11×10^{-2}. In the case of CIELab space, the sum of MSE's was 6.65×10^{-2}.

TABLE I. Results in terms of the MSE ($\times10^{-2}$) of the gaussian functions fitting to each one of the 11 colour categories considered. In the last column, the sum of the individual MSE's is presented as a global measure of the error of the model.

CATEGORY	Red	Orange	Brown	Yellow	Green	Blue	Purple	Pink	White	Grey	Black	Total
uvI Space	0.51	0.25	1.26	0.29	4.75	1.54	3.21	1.63	0.1	0.53	0.02	14.11
CIELab	0.25	0.12	0.59	0.24	2.34	0.69	1.55	0.61	0.1	0.16	0.00	6.65

Despite the global error of the model might seem not very high, several categories (brown, green, blue, purple and pink) are very badly modelled. Moreover, some facts indicate that the gaussian model is only adequate to model the achromatic categories (white, grey and black). If the intensity component is eliminated, then it is easy to show that the membership maps over the *uv*-plane derived from the experiment for the chromatic categories (red, orange, brown, yellow, green, blue, purple and pink), are not normal distributions (figure 3). Hence, this brought us to conclude that a different membership function for the chromatic categories could provide better results.

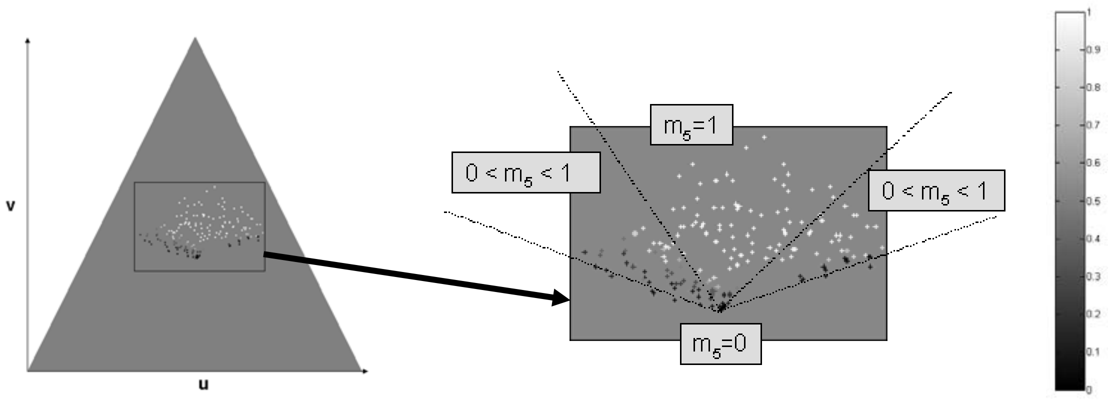

FIG. 3. Membership values for the green category (C_5) derived from the psycho-physical colour naming experiment. Each sample with green membership value (m_5) higher than zero is represented as a point in the diagram where grey level intensity indicates its certainty value to be named 'green'. Relationship between certainty and grey level is indicated in the scale on the right.

Sigmoid-gaussian model

After the first approach, we focussed on the modelling of the chromatic categories on the *uv*-plane. The introduction of the intensity in the model will be explained afterwards. The study of the membership maps over the *uv*-plane (figures 3 and 4) allowed us to define the desirable properties that should fulfil a characteristic function, $f_{C_k}(\underline{x})$ with $\underline{x}=(u,v)$, for the cited colour categories:

- $f_{C_k}(\underline{x}) \in [0,1]$, in order to be a membership function.
- $f_{C_k}(\underline{x})$ has a plateau form in its central area that spans on a triangular basis with a principal vertex.
- $f_{C_k}(\underline{x})$ is a parametrical function with parameters controlling the slope of the surface on the boundaries formed by the two sides of the triangular basis that share the principal vertex.
- $f_{C_k}(\underline{x})$ is a parametrical function with parameters allowing asymmetry with respect to the central axis, that is, the bisector of the angle formed by the two sides of the triangular basis that have the principal vertex in common.

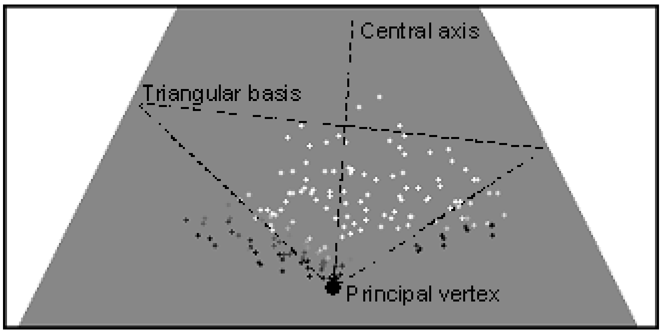

FIG. 4. Chromatic categories have a triangular basis with a principal vertex and a central axis. In the central area of the triangular basis, samples have membership values near one. In the areas near the two sides of the triangle that share the principal vertex membership values decrease from one to zero.

Several functions were considered to achieve the above constraints. However, the best results have been obtained by using a combination of two well-known functions. The first three constraints are fulfilled by using a sigmoid function. To reach the last one, we propose to modulate the sigmoid with a gaussian function in the direction perpendicular to the central axis of the category. To simplify the definition of this function we will suppose that we have all the points of the category being modelled in the first quadrant. To avoid confusions we will denote these axes as *u'v'* and will suppose that the central axis of the category is the line *u'=v'*.

The sigmoid function in one dimension is defined as:

$$S(x,\beta) = \frac{1}{1+e^{-\beta x}} \tag{8}$$

where β is a parameter that controls the slope of the function. Hence, we define the sigmoid function in two dimensions for our *u'v'*-plane as:

$$S(u',v',\beta_u,\beta_v) = \frac{1}{1+e^{-\beta_u u'}}\frac{1}{1+e^{-\beta_v v'}} \tag{9}$$

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Once the sigmoid function has been defined, we define the 1D-gaussian function which we have proposed to modulate the sigmoid function as:

$$G(u',v',\mu,\sigma) = e^{-\frac{1}{2}\left(\frac{\frac{u'-v'}{2^{1/2}}-\mu}{\sigma}\right)^2} \tag{10}$$

where μ is the mean and σ is the standard deviation. Figure 5 shows an scheme of the purpose of this function.

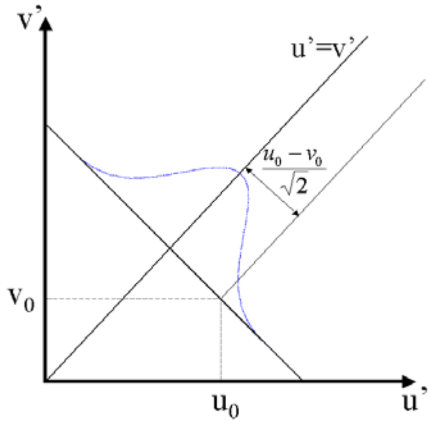
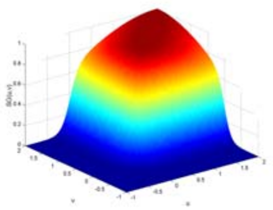


FIG. 5. A 1D-gaussian function is used to modulate the sigmoid in the direction *u'=v'*. For a point *(u_0,v_0)* the sigmoid function is modulated by the value of the gaussian at the position *(u_0 - v_0)/sqrt(2)*.

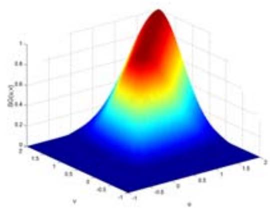
Hence, the final expression which will be used as characteristic function for each colour category is:

$$SG(u',v',\beta_u,\beta_v,\mu,\sigma) = \frac{1}{1+e^{-\beta_u u'}}\frac{1}{1+e^{-\beta_v v'}}e^{-\frac{1}{2}\left(\frac{\frac{u'-v'}{2^{1/2}}-\mu}{\sigma}\right)^2} \tag{11}$$

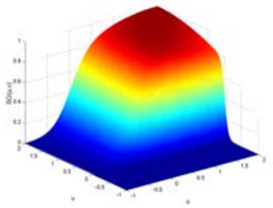
In figure 6, some examples of the sigmoid-gaussian function can be seen for different values of the parameters.



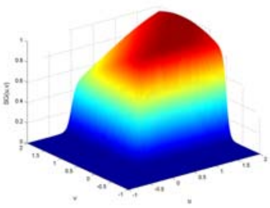
a) β_u=10, β_v=10, μ=0, σ=1.5



b) β_u=20, β_v=20, μ=0, σ=0.5



c) β_u=5, β_v=20, μ=0, σ=2



d) β_u=20, β_v=20, μ=0.5, σ=1.5

FIG. 6. Examples of the sigmoid-gaussian function for different parameters.

The function presented above has been defined supposing that the samples were on the positive quadrant of the space. Hence, the samples of the colour category to be modelled must be centred on the first quadrant. To this purpose, a translation $T(t_u,t_v)$ and a rotation $R(\alpha)$ are applied to the samples (equations 12 and 13). Notice the use of homogeneous coordinates which allow us to express both transformations as a matrix product.

$$\underline{x}' = R(\alpha)\cdot T(t_u,t_v)\cdot \underline{x} \tag{12}$$

$$\underline{x}' = \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -t_u \\ 0 & 1 & -t_v \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} (u-t_u)\cos\alpha - (v-t_v)\sin\alpha \\ (u-t_u)\sin\alpha + (v-t_v)\cos\alpha \\ 1 \end{pmatrix} \tag{13}$$

Hence, the final expression of the membership function, $f_{C_k}(\underline{x})$, used to model the eight chromatic colour categories has seven parameters:

$$f_{C_k}(\underline{x},\theta_k) = SG(u,v,(t_u,t_v,\alpha,\beta_u,\beta_v,\mu,\sigma)) = \frac{1}{1+e^{-\beta_u u'}}\,\frac{1}{1+e^{-\beta_v v'}}\,e^{-\frac{1}{2}\left(\frac{\frac{u'-v'}{2^{1/2}}-\mu}{\sigma}\right)^2} \tag{14}$$

As it has been shown, the sigmoid-gaussian function is defined on the *uv*-plane, that is, we only work using two dimensions. Previous works[10] and our first approaches with gaussian models have shown the usefulness of the intensity component for colour naming. Therefore, in order to consider the intensity component, we have divided the *uvI* colour space in three levels of intensity. For each level, all the samples inside it are represented and modelled on a 2D *uv*-plane (figure 7).

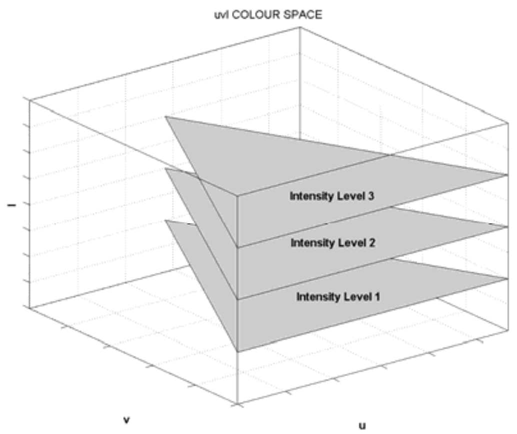


FIG. 7. Scheme of the division of the *uvI* colour space in three levels of intensity. All the samples inside a level of intensity are represented and modelled with the sigmoid-gaussian function on a 2D *uv*-plane.

The values that define the three levels were chosen in order to isolate some categories in only one or two of the intensity levels (i.e. yellow is only present for high intensity and orange does not appear for low intensity). Following this criterion and taking into account the results from the psycho-physical experiment, the values which provided the best results in the fitting process were chosen to divide the *uvI* space in three slices. These values for the *uvI* space were the 20% and 40% of the maximum intensity, that is, the intensity of the white of the model. This values are space-dependent and in the case of the CIELab space the values chosen were the 35% and the 70% of the maximum intensity.

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Modelling the achromatic categories

Due to their position in the *uv*-plane, and considering the previous results, the achromatic categories can be modelled with the gaussian model. However, we will need a different process for these categories (white, grey and black) which includes two steps. On the first step, the three categories are considered as a unique category and it is modelled with a 2D-gaussian function on the *uv*-plane for each intensity level. This will allow us to differentiate between the chromatic and the achromatic categories. On the second step, the membership values of the three achromatic categories are modelled as 1D-gaussian functions on the intensity axis. This will allow us to differentiate between white, grey and black once we have determined that we have an achromatic sample.

ESTIMATION OF A COLOUR NAMING MODEL

Once our sigmoid-gaussian colour naming model has been defined, the next step is to estimate the parameters for each characteristic function. In each intensity level, a function must be estimated for each one of the colour categories with samples in that intensity level. In figure 8, the samples for the three intensity levels are shown.

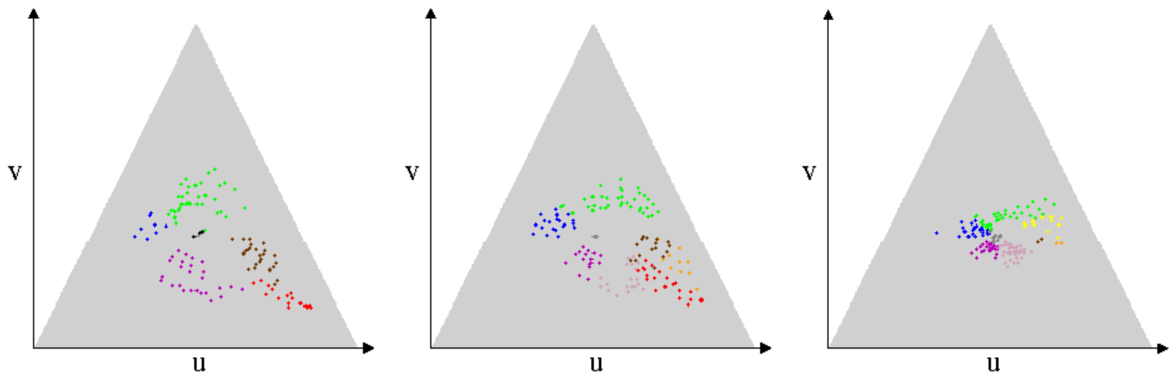

FIG. 8. Samples of the learning set for each intensity level. Left: Low intensity (Level 1). Centre: Medium intensity (Level 2). Right: High intensity (Level 3).

For a certain chromatic category C_k of an intensity level *p*, the modelling process works as follows. Firstly, all the samples of the learning set with intensity coordinate included in the level *p* are selected. Using the experiment membership values for category C_k, the seven parameters of the sigmoid-gaussian function are estimated. As in the case of the first gaussian approach, the parameter estimation is performed as a non-linear least-squares data fitting (equation 6).

On the other hand, the membership values of the three achromatic categories are summed and the parameters of the corresponding 2D multivariate gaussian functions are estimated with the same optimization algorithm as before. The final step is to estimate the parameters for the 1D gaussian functions across the intensity axis used to differentiate between the three achromatic categories (white, grey and black). The values of the parameters obtained on the *uvI* space as a result of the estimation process are presented in tables II to VI.

TABLE II. Model parameters ($\underline{\mu}$,Σ) for the achromatic 2D-gaussian functions with $\underline{\mu}=(\mu_1, \mu_2)$, $\Sigma_{1,1}=\sigma_0$, $\Sigma_{1,2}=\Sigma_{2,1}=\sigma_1$ and $\Sigma_{2,2}=\sigma_2$.

Intensity level	μ_1	μ_2	σ_0	σ_1	σ_2
Int < 60%	0.71	0.42	0.0032	0.0002	0.0006
60% < Int < 80%	0.72	0.42	0.0017	0.0001	0.0006
Int > 80%	0.73	0.42	0.0013	0.0002	0.0003

TABLE III. Model parameters for the 1D-gaussians modelling white, grey and black categories.

Colour	μ	σ
White	450.30	45.73
Grey	240.50	156.52
Black	16.06	26.21

TABLE IV. Model parameters for intensity level 1.

Colour	t_u	t_v	α	β_u	β_v	μ	σ
Red	0.83	0.37	1.09	49	53	0.110	0.04
Brown	0.78	0.33	0.38	140	119	0.080	0.13
Green	0.52	0.47	0.11	37	97	0.005	0.81
Blue	0.63	0.43	-2.55	118	49	0.050	0.14
Purple	0.64	0.44	2.21	178	107	0.050	0.25

TABLE V. Model parameters for intensity level 2.

Colour	t_u	t_v	α	β_u	β_v	μ	σ
Red	0.89	0.35	1.22	114	36	0.060	0.04
Orange	0.99	0.28	0.41	34	101	0.010	0.18
Brown	0.79	0.38	0.38	58	55	0.060	0.05
Green	0.54	0.50	0.11	91	75	0.100	0.34
Blue	0.66	0.46	-2.63	240	69	0.004	0.14
Purple	0.80	0.39	3.14	55	161	0.060	0.11
Pink	0.83	0.38	2.41	42	192	-0.050	0.12

TABLE VI. Model parameters for intensity level 3.

Colour	t_u	t_v	α	β_u	β_v	μ	σ
Orange	0.87	0.44	0.88	90	44	0.070	0.04
Brown	0.79	0.35	1.03	9	505	-0.060	0.03
Yellow	0.80	0.43	0.59	135	198	0.007	0.03
Green	0.67	0.44	-0.22	48	145	0.007	0.64
Blue	0.70	0.43	-2.27	200	65	0.001	0.05
Purple	0.74	0.40	3.14	407	227	-3.580	9.02
Pink	0.73	0.40	1.34	170	554	0.030	0.06

The results in terms of the MSE error for all the categories are presented in table VII. In this case, the total fitting error of the model on the *uvI* space is 1.98×10^{-2}. The fitting was repeated on the CIELab space, where the total error is 4.60×10^{-2}. On the *uvI* space, all the categories are modelled with less error than before and in some cases, such as in green or purple categories, the improvement is remarkable. On the CIELab space, the improvement is less evident but it is still important for some of the categories. These results confirm that the proposed sigmoid-gaussian model is modelling the colour categories better than the gaussian model.

TABLE VII. Results in terms of the MSE ($\times 10^{-2}$) of the sigmoid-gaussian functions fitting to each one of the 11 colour categories considered. In the last column, the sum of the individual MSE's is presented as a global measure of the error of the model.

CATEGORY	Red	Orange	Brown	Yellow	Green	Blue	Purple	Pink	White	Grey	Black	Total
uvI Space	0.12	0.16	0.52	0.20	0.28	0.08	0.16	0.17	0.04	0.25	0.0	1.98
CIELab	0.13	0.24	0.87	0.29	1.71	0.16	0.24	0.70	0.06	0.14	0.02	4.60

Once the parameters of the functions that characterize the fuzzy colour categories have been estimated, we are able to compute the colour descriptor $CD(\underline{x})$ for any sample $\underline{x}$ in the colour space. The information contained by $CD(\underline{x})$ can be used by a decision function $N(\underline{x})$ that assigns to $\underline{x}$ one of the 11 colour terms considered. At the moment, the decision function we have chosen assigns the colour term which corresponds to the category C_k with the highest membership value m_k in $CD(\underline{x})$:

$$N(\underline{x}) = t_k \mid m_k = \max_i \left\{ f_{C_i}(\underline{x}) \right\} \quad (15)$$

where t_k is the linguistic term corresponding to category C_k. Hence, to assign a colour name to a sample $\underline{x}$ in the colour space, the membership values for all the categories in the intensity level in which $\underline{x}$ is included are computed. These membership values form the colour descriptor $CD(\underline{x})$, which is normalized to sum one, and the decision function $N(\underline{x})$ is applied to obtain the colour name associated to $\underline{x}$.

The psychophysical experiment described above has provided us a set of membership values for all the samples in the learning set. Hence, the experiment results have been considered the target values for the output of the characteristic functions. Up to this point, the mean-squared error (MSE) between the values returned by the models and the ones obtained in the experiment has been used as a goodness measure of the model fitting to the colour data. However, as the main goal of a colour naming system is to automatically assign a colour term to a colour stimulus, we should also evaluate the validity of the model in terms of the number of samples which are correctly named.

The colour descriptors for all the samples in the learning set have been computed with the membership functions estimated and the decision function (equation 15) has been applied to obtain a set of automatic name assignments. This categorization has been compared to the one obtained from the psycho-physical experiment to compute the percentage of samples which are correctly named. The same has been done to the naming obtained by applying the first gaussian approach. The results are presented in table VIII.

TABLE VIII. Results in terms of MSE in the learning data fitting and number of samples correctly named.

Model	**Color Space**	**MSE**	**Samples correctly named**	**Samples correctly named (%)**
Gaussian	*uvI*	14.11×10^{-2}	370	87.68 %
Sigmoid-gaussian	*uvI*	1.98×10^{-2}	406	96.21 %
Gaussian	CIELab	6.65×10^{-2}	405	95.97 %
Sigmoid-gaussian	CIELab	4.60×10^{-2}	395	93.60 %

As can be seen in table VIII, the results obtained by the sigmoid-gaussian model on the *uvI* space are better than the ones obtained with the gaussian model, both in terms of the MSE in the data fitting process and the percentage of samples which are assigned the same name as the subjects in the psycho-physical experiment did. The improvement is specially important in the case of the MSE,

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since the error of 14.11×10^{-2} on the gaussian model is decreased to 1.98×10^{-2}. Moreover, the percentage of the learning samples which are correctly named increases to 96.21 %. This means that 406 of the 422 samples are assigned the same colour name than the mean subject of the experiment. Hence, the sigmoid-gaussian function considerably improves the results obtained by the gaussian model on this colour space.

On the CIELab space, the improvement of the results in terms of the MSE obtained by the Sigmoid-gaussian model is lower than in the *uvI* space, but the error decrease is still considerable (from 6.65×10^{-2} to 4.60×10^{-2}). When analysing, the results in terms of the number of samples which are correctly named it can be seen that the better results correspond to the gaussian model. However, as it will be seen in the next section, the gaussian model obtains poor results when it is tested over a wider data set different from the one used on the learning step. This means that although the gaussian model can describe the learning data set better than the sigmoid function, the whole colour space is better modelled by the sigmoid-gaussian model.

RESULTS

In the previous section, we have evaluated the fitting results of the estimation of our colour naming model. In this section, we present the results obtained when the sigmoid-gaussian model is used to categorize the Munsell colour array.

A set of Munsell reflectances were obtained from the Lappeenranta University of Technology (http://www.it.lut.fi/ip/research/color/database/download.html). These reflectances correspond to the stimuli used by Berlin and Kay for their anthropological studies, and were used to obtain their computational representation on the *uvI* colour space according to equations 2 to 5. The illuminant and the camera sensitivities used to generate this set of data were the same used to generate the learning set. The reflectances of ten samples of the Munsell array were not available and therefore we do not have their computational representations. These samples correspond to value 2 and hue from 5YR to 7.5GY.

Once we had the computational representation of the Munsell colour array, our model was applied to obtain the colour descriptor, $CD(\underline{x})$, for each chip of the Munsell array. Then, the decision function, $N(\underline{x})$, was applied and, finally, the colour term provided by $N(\underline{x})$ was assigned to the sample if the maximum membership value of $CD(\underline{x})$ was higher than a threshold value τ. In our case, we set τ=0.5 and considered that samples with a maximum membership value of less than 0.5 did not have a definite colour name. The categorization obtained is presented in figure 9, where each chip is painted with the colour corresponding to the assigned category. Inside each chip, the highest membership value obtained by our model is provided. Chips without a definite colour term, that is a highest membership value below τ, are coloured in light blue. The chips in dotted pattern correspond to the samples that were not available.

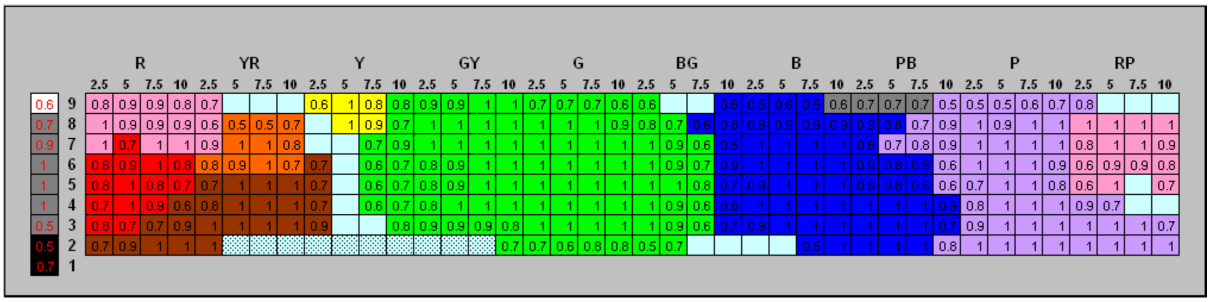

FIG. 9. Categorization of the Munsell colour array obtained by applying our proposed sigmoid-gaussian model on the *uvI* space.

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The categorization of the Munsell array has also been obtained by applying the gaussian and the sigmoid gaussian model on the CIELab space (figures 10 and 11). The categorization obtained by the gaussian model on the *uvI* space was very poor and will not be shown here.

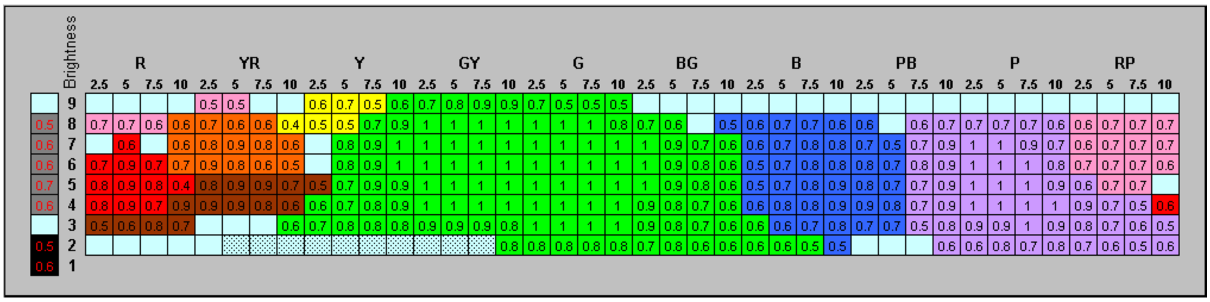

FIG. 10. Categorization of the Munsell colour array obtained by applying the gaussian model on the CIELab space.

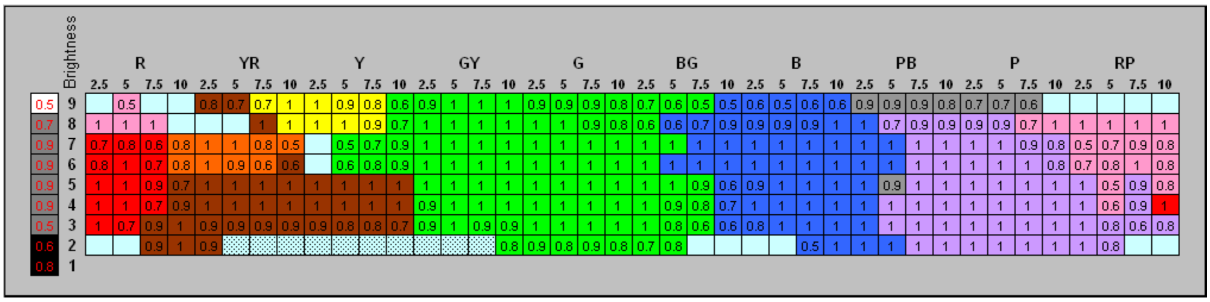

FIG. 11. Categorization of the Munsell colour array obtained by applying our proposed sigmoid-gaussian model on the CIELab space.

The categorization of the Munsell array obtained by our model was compared to the results obtained in previous works about colour naming. The comparison was done to the boundaries for American English derived from Berlin and Kay experiments (figure 12) and to the categorization done by a 35 year old male English speaker presented by MacLaury[26] (figure 13). At this point we have to remark that although the subjects of our experiment were Catalan and/or Spanish speakers, we compare our results to the ones obtained from English speakers. This fact is due to the lack of previous results about these two languages. Therefore, some of the differences between our categorization and the others could be due to this fact. Finally, our results are compared with the previous approach of Lammens to automatic colour naming for computer vision (figure 14).

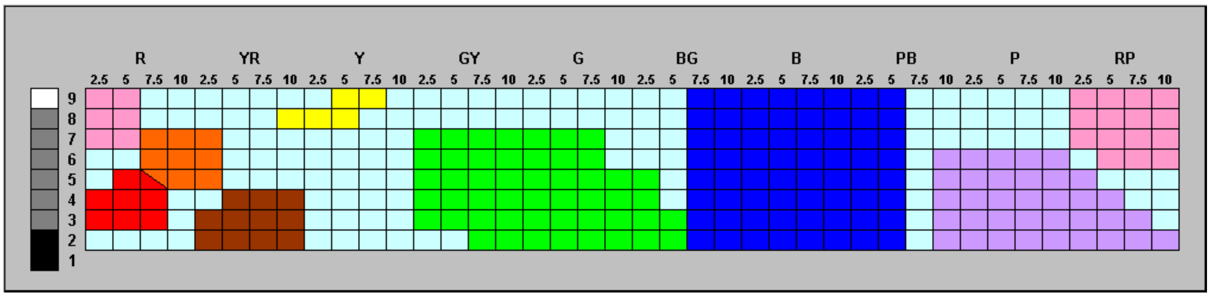

FIG. 12. Categorization of the Munsell colour array obtained by Berlin and Kay in their experiments for American English.

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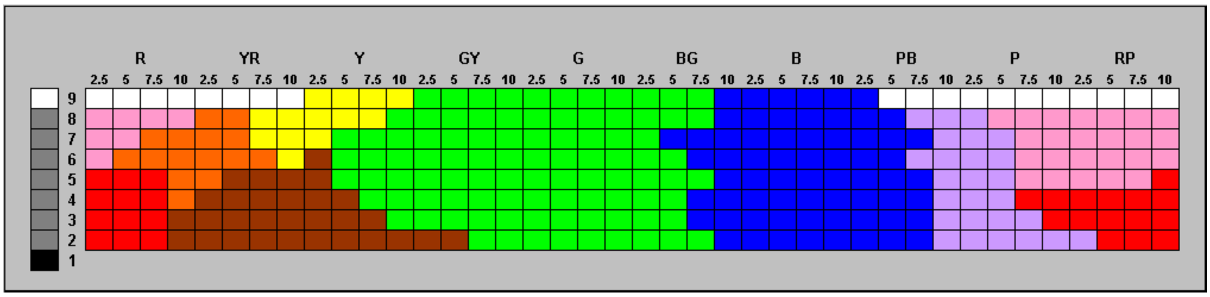

FIG. 13. Categorization of the Munsell colour array provided by MacLaury's English speaker[26].

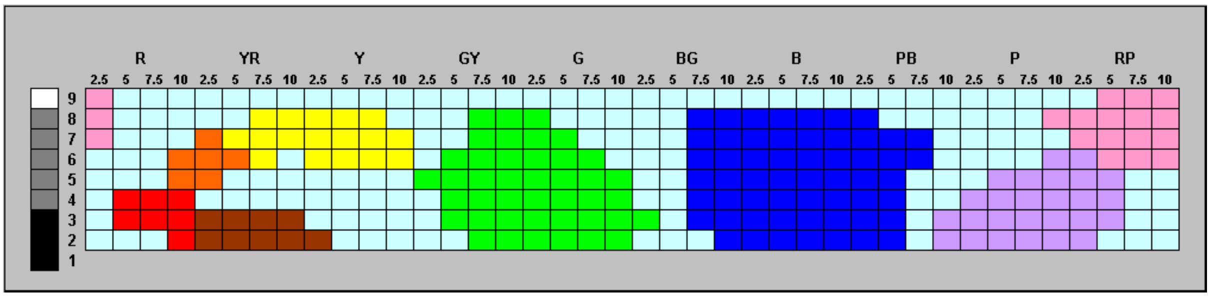

FIG. 14. Results of Lammens gaussian model on the CIELab colour space which is the one where the model obtained the best results.

The comparison of our model's categorization with the results of Berlin and Kay studies and the categorization made by MacLaury's English speaker shows that our categorization considerably agrees with these previous ones. It must be remarked that both categorizations were made by human observers which means that the output of our model approaches the goal of obtaining a behaviour similar to that of a human being. Apart from small differences in the categorization of single chips, the most important difference is found on the orange category. In our results, orange is shifted compared to Berlin and Kay experiments. However, the observation of the Munsell samples, the learning samples and the subject's responses in the experiment have brought us to consider the possibility that this is a cultural difference, since the results of the model agree with the subjects judgements on the experiment. At this point, it is easy to see the subjectivity of the problem, since the categorization of MacLaury's English speaker has very important differences with Berlin and Kay division of the colour space. See, for example, that red in the subject's categorization overlaps the purple region in Berlin and Kay results.

To have an objective measure of the performance of our model, we considered the chips inside the boundaries proposed by Berlin and Kay (210 of the 329 chips on the Munsell array) and evaluated the number of coincidences and errors of the other categorizations compared with Berlin and Kay results. To do this, we calculated the number of chips inside the boundaries which are named with the same term in the two categorizations being compared. The results show that the number of coincidences and errors in our categorization on the *uvI* space and MacLaury's English speaker is the same while the percentage of coincidences in Lammens results is considerably lower. The results of our model are even better on the CIELab space. Hence, this evaluation confirms that our model has a behaviour similar to that of human observers, and that our model improves the results obtained by the previous computational approach of Lammens. These results are summarized in table IX.

TABLE IX. Comparison of the different categorizations in terms of the coincidence with Berlin and Kay categorization.

Categorization	Coincidences	Errors	% Coincidences
Lammens gaussian model	161	49	76.67 %
Gaussian model (CIELab)	173	37	82.38 %
MacLaury's English speaker	182	28	86.67 %
Sigmoid-gaussian model (*uvI* space)	182	28	86.67 %
Sigmoid-gaussian model (CIELab)	188	22	89.52 %

Another important advantage of our model is that the membership values are expanded in a quite perceptual sense, since values of one are in the centre of the categories and these values decrease as we approach to the boundaries of the category, where they tend to the threshold value. Despite the good results, we have noticed that there are some errors on the sigmoid-gaussian categorization which are concentred in the extremes of the intensity scale, that is, the samples with low and high value in the Munsell system. This problem is due to some deficiencies of the learning set with these samples that, in some cases, were unavailable. However, despite this considerations, the categorization provided by our model is very coherent with human judgements and the results show the validity of the sigmoid-gaussian function to model the colour naming categories.

CONCLUSIONS

In this paper we have proposed a general model of colour naming in digital images, which can be applied to real computer vision problems. The model is based on fuzzy set theory and each colour category has a characteristic function which provides a membership value to that category for any sample in the colour space. The full model has been defined and estimated.

The model presents the eight chromatic categories defined by Berlin and Kay as basic colour terms modelled with a sigmoid-gaussian function. This function fulfils a set of desirable properties that colour naming characteristic functions should have. The three achromatic categories are modelled using gaussian functions and intensity is used to differentiate between them. The functions for all the categories have been fitted to data from a psycho-physical experiment with a low MSE. The model has also been tested on the Munsell colour array and the results support the validity of our model to address the colour naming problem. Although there are still some problems, our model can be a first useful step on the way of automating the colour naming task.

As future research lines, we firstly should consider the colour constancy problem[27] and take into account the effects that a change in the illumination might have over the final result of the colour naming. Secondly, the results obtained could be improved by using the knowledge we have about the structure of the colour naming space. This means, that some constraints should be applied to the process in order to obtain a better estimation of the parameters of the membership functions. Finally, we should study the performance of the model when the acquisition device is changed in order to evaluate the validity of the model for different device-dependent colour spaces.

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